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Higher order contributions to the effective action of $\mathcal{N} = 2$ super Yang-Mills

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Abstract: We apply heat kernel techniques in $\mathcal{N} = 1$ superspace to compute the one-loop effective action to order F^5 for chiral superfields coupled to a non-abelian super Yang-Mills background. The results, when combined with those of [4], yield the one-loop effective action to order F^5 for any $\mathcal{N} = 2$ super Yang-Mills theory coupled to matter hypermultiplets.

Keywords: AdS-CFT and dS-CFT Correspondence, Supersymmetric Effective Theories, Superspaces.

Contents

1. Introduction

The low energy effective actions for D-branes in superstring theories are supersymmetric Yang-Mills theories. This fact has led to a remarkably fruitful interface between superstring theory and supersymmetric Yang-Mills theory, which has yielded valuable insights on both sides. Perhaps the most spectacular example is the Maldacena conjecture of the duality between $\mathcal{N} = 4$ supersymmetric Yang-Mills theory and type-IIB superstring theory in an $AdS_5 \times S^5$ background [1, 2, 3]. On the field theory side, more stringent tests of this conjecture require the computation of the effective action of $\mathcal{N} = 4$ supersymmetric Yang-Mills theory. In [4], the one-loop effective action for $\mathcal{N} = 4$ supersymmetric Yang-Mills theory was computed through to order F^6 using the $\mathcal{N} = 1$ superfield formulation. At the component level the results to order F^5 were found to be in agreement with string theory calculations to the same order [6] (see also [5]).

Maldacena's original arguments have been generalized to include a conjectured duality between certain $\mathcal{N} = 2$ superconformal supersymmetric Yang-Mills theories and superstring theories in special backgrounds [7, 8]. As in the $\mathcal{N} = 4$ case, more detailed tests of these conjectures will require comparison of the effective action for specific $\mathcal{N} = 2$ supersymmetric Yang-Mills theories with results derived from superstring calculations. In anticipation of such tests, it is important that the effective action of these supersymmetric Yang-Mills theories be computed to higher orders. This paper details the computation of the one-loop effective action for general $\mathcal{N} = 2$ supersymmetric Yang-Mills theories to order F^5 in $\mathcal{N} = 1$ superfield formulation. Since at the quantum level, with the same gauge conditions, $\mathcal{N} = 2$ and $\mathcal{N} = 4$ super Yang-Mills theories differ only in the hypermultiplet sector, making use of the results already derived in [4], we need only calculate the one-loop contribution associated with this sector.

This paper is organized as follows. Firstly, in section 2, we briefly discuss the connection between $\mathcal{N} = 2$ and $\mathcal{N} = 4$ super Yang-Mills theories at the classical and one-loop quantum level. In section 3 we review, using heat kernels and zeta functional regularization, the quantization of matter coupled to a non-abelian super Yang-Mills background in $\mathcal{N} = 1$ superspace. We also demonstrate how the one-loop effective action may be cast on full superspace, rather than a chiral subspace. Section 4 briefly describes the method of computation of the associated heat kernel, being further explained in appendix A. The results of the calculation in full superfield form, as well as its leading bosonic component, are given to order F^5 in section 5. Section 6 compares these results with partial bosonic results currently available in the literature, and section 7 discusses the one-loop effective action for general $\mathcal{N} = 2$ super Yang-Mills theory to this order. Appendix B details the change of basis calculations required to compare the F^5 component results with literature.

We adopt the conventions and notation of [9] and [10].

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2. $\mathcal{N} = 2$ super Yang-Mills

The most general classical non-abelian $\mathcal{N} = 2$ supersymmetric Yang-Mills action cast in $\mathcal{N} = 1$ superfield form consists of two parts,

$$S_{\mathcal{N}=2} = S_{\text{pure}} + S_{\text{hyper}} \,. \tag{2.1}$$

The pure $\mathcal{N} = 2$ super Yang-Mills action is given by:

$$S_{\text{pure}} = \frac{1}{g^2}\text{tr}_{\text{Ad}}\left(\int \mathrm{d}^8 z\ \Phi^\dagger \Phi + \frac{1}{2}\int \mathrm{d}^6 z\ W^2\right), \tag{2.2}$$

where Ad denotes the adjoint representation. The hypermultiplet action is

$$\begin{aligned} S_{\text{hyper}} = &\int \mathrm{d}^8 z \big(Q^\dagger Q + \tilde{Q}^\dagger \tilde{Q}\big) + \sqrt{2}\int \mathrm{d}^6 z\ \tilde{Q}^{\mathrm{T}} \Phi\, Q + \sqrt{2}\int \mathrm{d}^6 \bar{z}\ Q^\dagger \Phi^\dagger \bar{\tilde{Q}} + \\ &+ \mathcal{M}\int \mathrm{d}^6 z\ \tilde{Q}^{\mathrm{T}}\, Q + \mathcal{M}\int \mathrm{d}^6 \bar{z}\ Q^\dagger \bar{\tilde{Q}} \end{aligned} \tag{2.3}$$

where $\mathcal{M}$ is the hypermultiplet mass. The covariantly chiral superfields Q, $\tilde{Q}$ and Φ transform in the representations R, its conjugate R_c, and the adjoint representation of the gauge group respectively. W^α is the covariantly chiral superfield strength associated with the gauge covariant derivatives,

$$\mathcal{D}_A = (\mathcal{D}_a, \mathcal{D}_\alpha, \bar{\mathcal{D}}^{\dot\alpha}) = D_A + \mathrm{i}\Gamma_A\,, \qquad \Gamma_A = \Gamma_A^I(z) T^I\,, \tag{2.4}$$
$$(T^I)^\dagger = T^I\,, \qquad [T^I, T^J] = \mathrm{i} f^{IJK} T^K\,, \tag{2.5}$$

where D_A are flat covariant derivatives and T^I are the hermitean generators of the gauge group. The gauge covariant derivatives satisfy the algebra:

$$\begin{aligned}
\{\mathcal{D}_\alpha, \mathcal{D}_\beta\} &= \{\bar{\mathcal{D}}_{\dot\alpha}, \bar{\mathcal{D}}_{\dot\beta}\} = 0 \\
\{\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot\alpha}\} &= -2\mathrm{i}\mathcal{D}_{\alpha\dot\alpha} = -2\mathrm{i}(\sigma^a)_{\alpha\dot\alpha}\mathcal{D}_a \\
[\mathcal{D}_\alpha, \mathcal{D}_{\beta\dot\beta}] &= 2\mathrm{i}\varepsilon_{\alpha\beta}\bar{W}_{\dot\beta} \\
[\bar{\mathcal{D}}_{\dot\alpha}, \mathcal{D}_{\beta\dot\beta}] &= 2\mathrm{i}\varepsilon_{\dot\alpha\dot\beta}W_\beta \\
[\mathcal{D}_{\alpha\dot\alpha}, \mathcal{D}_{\beta\dot\beta}] &= (\sigma^a)_{\alpha\dot\alpha}(\sigma^b)_{\beta\dot\beta}G_{ab} = -\varepsilon_{\alpha\beta}(\bar{\mathcal{D}}_{\dot\alpha}\bar{W}_{\dot\beta}) - \varepsilon_{\dot\alpha\dot\beta}(\mathcal{D}_\alpha W_\beta)\,.
\end{aligned} \tag{2.6}$$

We are interested in the one-loop effective action for a generic $\mathcal{N}=2$ SYM theory (2.1), which can be computed efficiently using the $\mathcal{N}=1$ background field formalism [11, 12]. Here we consider providing only the $\mathcal{N}=1$ vector multiplet with background field values. After background covariant gauge fixing [12, 13], the effective action at the one-loop level is a linear combination of two types of contribution [11, 14] - the logarithm of the superdeterminant of the background covariant operator in the quadratic part of the action for a vector superfield in the adjoint representation, and the logarithm of the superdeterminant of the operator which results from the quadratic part of the action for a background covariantly chiral scalar superfield in a real representation of the gauge group (either the adjoint representation or $R \oplus R_c$). Since the former has already been dealt with in [4], it remains to compute the latter in order to be able to assemble the one-loop low energy effective action for an arbitrary $\mathcal{N}=2$ SYM theory. For related discussions, material and different approaches to analogous problems also see [15]–[25].

3. Chiral superfields in a super Yang-Mills background

As discussed above, the problem of computing the one-loop effective action of any $\mathcal{N}=2$ SYM theory is reduced to computing the effective action for a background covariantly chiral scalar $\chi = \{\chi^i(z)\}$, which transforms under some real representation $\mathcal{R}$ of the gauge group. The background covariant derivatives are given by

$$\mathcal{D}_A = (\mathcal{D}_a, \mathcal{D}_\alpha, \bar{\mathcal{D}}^{\dot\alpha}) = D_A + \mathrm{i}\Gamma_A\,, \qquad \Gamma_A = \Gamma_A^I(z)\tilde{T}^I\,, \tag{3.1}$$

and satisfy the algebra (2.6), where here the hermitean generators $(\tilde{T}^I)^i{}_j$ are antisymmetric:

$$(\tilde{T}^I)^\dagger = \tilde{T}^I \qquad (\tilde{T}^I)^{\mathrm{T}} = -\tilde{T}^I\,, \tag{3.2}$$

Since we are interested only in one-loop corrections, any interactions may be ignored, and the classical action in question is:

$$S[\chi,\bar\chi] = \int \mathrm{d}^8 z\, \chi^\dagger \chi + \frac{m}{2}\Big\{\int \mathrm{d}^6 z\, \chi^{\mathrm{T}}\chi + \mathrm{c.c.}\Big\}\,. \tag{3.3}$$

The mass m is either an explicit mass (as in the case of the hypermultiplet mass $\mathcal{M}$ in (2.3)), or a infrared regulator for massless chiral scalars (as in the case of Φ in (2.3) and the ghosts)[1].

One can efficiently calculate the one-loop contribution to the effective action via zeta function regularization, and it is proportional to $\zeta'(0)$, where the zeta function is defined by

$$\zeta(s) = \frac{\mu^{2s}}{\Gamma(s)} \int_0^\infty \mathrm{d}t\, t^{s-1} e^{-tm^2} \big(K_+(t) + K_-(t)\big) . \tag{3.4}$$

In this expression, μ is the renormalization point and $K_+(t)$ and $K_-(t)$ are the functional traces of the chiral and antichiral heat kernels respectively, which are defined by:

$$K_\pm(t) = \mathrm{tr}_\mathcal{R} \int \mathrm{d}^6 z_\pm \int \mathrm{d}^6 z'_\pm \delta_\pm(z,z')\ e^{t\Box_\pm} \delta_\pm(z,z') \equiv \mathrm{tr}_\mathcal{R} \int \mathrm{d}^6 z_\pm\, K_\pm(z,t) . \tag{3.5}$$

Here $\mathrm{tr}_\mathcal{R}$ denotes the trace over the representation $\mathcal{R}$, $\mathrm{d}z_\pm$ the integration measure over (anti)chiral subspace, $\delta_\pm(z,z')$ the (anti)chiral delta functions,

$$\delta_+(z,z') = -\frac{1}{4}\bar{\mathcal{D}}^2 \mathbb{I}\, \delta^{(8)}(z,z') \tag{3.6}$$

$$\delta_-(z,z') = -\frac{1}{4}\mathcal{D}^2 \mathbb{I}\, \delta^{(8)}(z,z') \tag{3.7}$$

$$\delta^{(8)}(z,z') = \delta^{(4)}(x,x')\delta^{(2)}(\theta-\theta')\delta^{(2)}(\bar\theta-\bar\theta') , \tag{3.8}$$

and

$$\Box_+ = \frac{1}{16}\bar{\mathcal{D}}^2\mathcal{D}^2 \tag{3.9}$$

$$\Box_- = \frac{1}{16}\mathcal{D}^2\bar{\mathcal{D}}^2 . \tag{3.10}$$

It can be shown that

$$K_+(t) = K_-(t) \tag{3.11}$$

and so

$$\zeta(s) = \frac{2\mu^{2s}}{\Gamma(s)} \int_0^\infty \mathrm{d}t\, t^{s-1} e^{-tm^2} K_+(t) , \tag{3.12}$$

requiring computation of only the chiral kernel.

The operator

$$\Box_+ = \frac{1}{16}\bar{\mathcal{D}}^2\mathcal{D}^2 , \tag{3.13}$$

is a Laplace-type operator when acting on chiral superfields, Ψ,

$$\Box_+ \Psi = (\mathcal{D}^a\mathcal{D}_a - W^\alpha\mathcal{D}_\alpha - \frac{1}{2}(\mathcal{D}^\alpha W_\alpha))\Psi . \tag{3.14}$$

This can be established using the identity

$$\mathcal{D}^a\mathcal{D}_a - W^\alpha\mathcal{D}_\alpha = -\frac{1}{8}\bar{\mathcal{D}}_{\dot\alpha}\mathcal{D}^2\bar{\mathcal{D}}^{\dot\alpha} + \frac{1}{2}(\mathcal{D}^\alpha W_\alpha) + \frac{1}{16}(\mathcal{D}^2\bar{\mathcal{D}}^2 + \bar{\mathcal{D}}^2\mathcal{D}^2) . \tag{3.15}$$

[1]One may also introduce a background for the adjoint scalar Φ in (2.1) taking values in the Cartan subalgebra, which then generates masses for the vector multiplet, the hypermultiplet and ghosts. For a discussion see [25].

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To compute the effective action, it suffices to consider an on-shell background, $\mathcal{D}^\alpha W_\alpha = \bar{\mathcal{D}}_{\dot\alpha}\bar{W}^{\dot\alpha} = 0$, so that

$$\Box_+ = \frac{1}{16}\bar{\mathcal{D}}^2\mathcal{D}^2 = \mathcal{D}^a\mathcal{D}_a - W^\alpha\mathcal{D}_\alpha \tag{3.16}$$

acting on chiral superfields.

$K_+(z,t)$ has an asymptotic expansion in t in the limit $t \to 0$, which is usually expressed

$$K_+(z,t) = \frac{1}{16\pi^2 t^2}\sum_{n=0}^{\infty} t^n a_n(z)\,, \qquad a_0 = a_1 = 0\,. \tag{3.17}$$

The $a_n(z)$ are the DeWitt-Seeley coefficients, and are chiral superfields which at the component level contain bosonic field strength terms of the form F^n. To the best of our knowledge only the first non-trivial coefficient a_2 is known in the non-abelian case [27]:

$$a_2 = W^2\,. \tag{3.18}$$

Evaluating the one-loop effective action, $\Gamma^{(1)}_{\chi,\mathcal{R}}$, therefore amounts to computing the DeWitt-Seeley coefficients:

$$\Gamma^{(1)}_{\chi,\mathcal{R}} = \frac{1}{4}\zeta'(0) = \frac{1}{16\pi^2}\ln\left(\frac{\mu}{m}\right)\int \mathrm{d}^6 z\, \mathrm{tr}_{\mathcal{R}}(a_2) + \frac{1}{32\pi^2}\sum_{n=3}^{\infty}\frac{(n-3)!}{m^{2n-4}}\int \mathrm{d}^6 z\, \mathrm{tr}_{\mathcal{R}}(a_n)\,. \tag{3.19}$$

It turns out that the a_n with $n \geq 3$ are expressible as $\bar{\mathcal{D}}^2$ acting on field strengths and their covariant derivatives, and so the second term on the right hand side of (3.19) can be lifted to a gauge-invariant superfunctional on full superspace. This can be proven as follows.

By differentiating the kernel $K_+(z,t)$ with respect to t, one observes that:

$$\begin{aligned}
\frac{dK_+(z,t)}{dt} &= \frac{1}{16}\int \mathrm{d}^6 z'_+\,\delta_+(z,z')\bar{\mathcal{D}}^2\mathcal{D}^2\mathrm{e}^{t\Box_+}\delta_+(z,z') \\
&= \frac{1}{16}\bar{\mathcal{D}}^2\left(\int \mathrm{d}^6 z'_+\,\delta_+(z,z')\mathcal{D}^2\mathrm{e}^{t\Box_+}\delta_+(z,z')\right) \\
&= \frac{1}{16}\bar{\mathcal{D}}^2\left(\lim_{z'\to z}\mathcal{D}^2\mathrm{e}^{t\Box_+}\delta_+(z,z')\right)\,,
\end{aligned} \tag{3.20}$$

since

$$\bar{\mathcal{D}}_{\dot\alpha}\delta_+(z,z') = 0\,. \tag{3.21}$$

On the other hand, (3.17) yields

$$\frac{dK_+(z,t)}{dt} = \frac{1}{16\pi^2}\sum_{n=3}^{\infty}(n-2)t^{n-3}a_n(z)\,. \tag{3.22}$$

Comparison of (3.20) and (3.22) demonstrates that the DeWitt-Seeley coefficients other than a_2 are expressible in the desired form.

It is convenient to introduce a new set of coefficients by writing $\lim_{z'\to z} \mathcal{D}^2 e^{t\Box_+} \delta_+(z,z')$ as an asymptotic series,

$$\lim_{z'\to z} \mathcal{D}^2 e^{t\Box_+} \delta_+(z,z') = \frac{1}{16\pi^2 t^2} \sum_{n=0}^{\infty} t^n c_n(z) \,, \tag{3.23}$$

where simple computation reveals

$$c_0 = -4\,\mathbb{I}\,, \qquad c_1 = 0\,, \tag{3.24}$$

whilst comparison of (3.20), (3.22) and (3.23) yields

$$a_n(z) = \frac{1}{16(n-2)} \bar{\mathcal{D}}^2(c_{n-1}(z)) \qquad n \geq 3\,. \tag{3.25}$$

The effective action can then be written as

$$\Gamma^{(1)}[V] = \frac{1}{16\pi^2} \ln\left(\frac{\mu}{m}\right) \int d^6 z \, \mathrm{tr}_{\mathcal{R}}(W^2) - \frac{1}{128\pi^2} \sum_{n=3}^{\infty} \frac{(n-3)!}{(n-2)m^{2n-4}} \int d^8 z \, \mathrm{tr}_{\mathcal{R}}(c_{n-1})\,, \tag{3.26}$$

the second term now being expressed in full superspace.

Consequently, determining the effective action reduces to computing the new coefficients c_n, which can of course be obtained by the same techniques used for computing DeWitt-Seeley coefficients. If desired, the DeWitt-Seeley coefficients themselves can be recovered through identity (3.25), which is nothing more than a projection onto the chiral subspace.

4. The method of computation

We wish to compute the effective action to order F^5, which corresponds to evaluating c_2, c_3 and c_4. The traditional method for computing heat kernel coefficients is the iterative DeWitt method (for example see [10]). We however prefer to use the method developed in [28, 29] and modified for non-abelian backgrounds in [4] since this calculation parallels that of [4], with only minor modifications for application to chiral subspace.

We begin by introducing a plane wave basis for the chiral delta function[2],

$$\delta_+(z,z') = 4\mathbb{I} \int \frac{d^4 k}{(2\pi)^4} \, e^{ik^a \omega_a} \int d^2\epsilon \; e^{i\epsilon^\alpha (\theta-\theta')_\alpha} \tag{4.1}$$

where[3]

$$\omega_a = x_a - x'_a - i\theta\sigma_a\bar{\theta}' + i\theta'\sigma_a\bar{\theta}\,. \tag{4.2}$$

In the coincidence limit, $\mathcal{D}^2 e^{t\Box_+}\delta_+(z,z')$ becomes

$$\lim_{z'\to z} \mathcal{D}^2 e^{t\Box_+} \delta_+(z,z') = K^\alpha{}_\alpha(z,t) = \int d\eta_+ \; X^\alpha X_\alpha e^{t\Delta}\,, \tag{4.3}$$

[2]From here onward we work in the chiral representation.

[3]Note that although ω_a is not itself chiral, $\bar{\mathcal{D}}_{\dot\alpha}(\omega_a) = -i(\sigma_a)_{\alpha\dot\alpha}(\theta-\theta')^\alpha$, the entire delta function is annihilated by $\bar{\mathcal{D}}_{\dot\alpha}$ since $(\theta-\theta')^3 = 0$.

the X's being defined by

$$X_a = \mathcal{D}_a + \mathrm{i}k_a \tag{4.4}$$
$$X_\alpha = \mathcal{D}_\alpha + \mathrm{i}\epsilon_\alpha \,, \tag{4.5}$$

and where the notation

$$K_{A_1 A_2 \dots A_n}(z,t) = \int \mathrm{d}\eta_+ \, X_{A_1} X_{A_2} \dots X_{A_n} e^{t\Delta} \tag{4.6}$$
$$\int \mathrm{d}\eta_+ = 4 \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \int \mathrm{d}^2\epsilon \tag{4.7}$$
$$\Delta = X^a X_a - W^\alpha X_\alpha \tag{4.8}$$

has been introduced. Note that there is also a shift $-k_{\alpha\dot{\alpha}}(\bar{\theta} - \bar{\theta}')^{\dot{\alpha}}$ in $\mathcal{D}_\alpha$ which always vanishes in the coincidence limit since there are no $\bar{\mathcal{D}}_{\dot{\alpha}}$ operators present. The X's satisfy the algebra

$$\{X_\alpha, X_\beta\} = 0 \,, \qquad [X_a, X_b] = G_{ab} \,, \qquad [X_\alpha, X_a] = \mathrm{i}(\sigma_a)_{\alpha\dot{\alpha}} \bar{W}^{\dot{\alpha}} \,. \tag{4.9}$$

In this notation the power series (3.23) is:

$$K^\alpha{}_\alpha(z,t) = \frac{1}{16\pi^2 t^2} \sum_{n=0}^{\infty} t^n c_n(z) \,. \tag{4.10}$$

Differentiating $K^\alpha{}_\alpha(z,t)$ with respect to t yields the differential equation

$$\frac{dK^\alpha{}_\alpha(z,t)}{dt} = K^\alpha{}_\alpha{}^a{}_a(z,t) \,. \tag{4.11}$$

Using the identities

$$0 = \int \mathrm{d}\eta_+ \, \frac{\partial}{\partial k_b} \left(X_\alpha X_\beta X_a \mathrm{e}^{t\Delta} \right) \tag{4.12}$$

and

$$[A, \mathrm{e}^B] = \int_0^1 \mathrm{d}s \; \mathrm{e}^{sB} [A,B] \mathrm{e}^{(1-s)B} \,, \tag{4.13}$$

it follows that

$$0 = \mathrm{i}\delta_a^b K_{\alpha\beta}(z,t) + 2\mathrm{i}t \int \mathrm{d}\eta_+ \; X_\alpha X_\beta X_a \sum_{n=0}^{\infty} \frac{t^n}{(n+1)!} ad_\Delta^n(X^b) \; \mathrm{e}^{t\Delta} \tag{4.14}$$

where ad^n denotes n nested commutators:

$$ad_A^0(B) = B \,, \qquad ad_A^n(B) = [A, \, ad_A^{n-1}(B)] \,. \tag{4.15}$$

After contraction of indices, this becomes

$$K^\alpha{}_\alpha{}^a{}_a(z,t) = -\frac{2}{t} K^\alpha{}_\alpha(z,t) - \int \mathrm{d}\eta_+ \; X^\alpha X_\alpha X^a \sum_{n=1}^{\infty} \frac{t^n}{(n+1)!} ad_\Delta^n(X_a) \; \mathrm{e}^{t\Delta} \,. \tag{4.16}$$

Inserting this into the differential equation (4.11), one obtains:

$$\frac{dK^{\alpha}{}_{\alpha}(z,t)}{dt}+\frac{2}{t}K^{\alpha}{}_{\alpha}(z,t)=-\int \mathrm{d}\eta_{+}\; X^{\alpha}X_{\alpha}X^{a}\sum_{n=1}^{\infty}\frac{t^{n}}{(n+1)!}ad^{n}_{\Delta}(X_a)\,\mathrm{e}^{t\Delta}\,, \tag{4.17}$$

the significance of which is seen in terms of the expansion (4.10), where the left hand side is

$$\frac{dK^{\alpha}{}_{\alpha}(z,t)}{dt}+\frac{2}{t}K^{\alpha}{}_{\alpha}(z,t)=\frac{1}{16\pi^2}\sum_{n=0}^{\infty}nt^{n-3}c_n(z)=\frac{c_1(z)}{16\pi^2t^2}+\frac{2c_2(z)}{16\pi^2 t}+\frac{3c_3(z)}{16\pi^2}+\cdots \tag{4.18}$$

As is usually the case with this approach, the differential equation yields an expansion where the first non-trivial coefficient c_0 is absent. The objective now becomes to determine the coefficients $c_n(z)$ by expanding the right hand side of (4.17) in a power series in t, and identifying it with the right hand side of (4.18).

Since the summation in (4.17) involves the repetitive calculation of commutators, it is first useful to establish the following relations:

$$\begin{aligned}
[\Delta, X_a] &= 2G^{b}{}_{a}X_b+(\mathcal{D}_aW^{\alpha})X_{\alpha}+\mathrm{i}(\sigma_a)_{\alpha\dot\alpha}\bar W^{\dot\alpha}W^{\alpha}\\
[\Delta, X_{\alpha}] &= (\mathcal{D}_{\alpha}W^{\beta})X_{\beta}-2\mathrm{i}(\sigma^a)_{\alpha\dot\alpha}\bar W^{\dot\alpha}X_a\\
[\Delta, A] &= (\mathcal{D}^a\mathcal{D}_aA)+2(\mathcal{D}^aA)X_a-W^{\alpha}(\mathcal{D}_{\alpha}A)-(-1)^{\varepsilon(A)}[W^{\alpha},A\}X_{\alpha}\,.
\end{aligned} \tag{4.19}$$

From these it is clear that summation will generate a series of moments of the form $K_{A_1\ldots A_i}(z,t)$ as defined in (4.6). Furthermore, it is not difficult to show that to order n in this summation, the moments generated have at most $(n+3)$ indices. It is convenient to always place these indices in a specific order: first spinor, then vector. This can be achieved through the commutation relations (2.6). With such an ordering, the leading term in a moment's asymptotic power series has the following behaviour[4]:

$$K_{A_1\ldots A_{q+p}}(z,t)\sim\frac{1}{t^2}\left(\frac{1}{t}\right)^{[\frac{p}{2}]}t^{2-q}=t^{-q-[\frac{p}{2}]}\qquad q\le 2 \tag{4.20}$$

where $K_{A_1\ldots A_{q+p}}(z,t)$ has q undotted spinor indices, p vector indices and $[\frac{p}{2}]$ denotes the largest integer part of $p/2$. Moments with greater than two undotted spinor indices vanish since $X_{\alpha}X_{\beta}X_{\gamma}=0$.

From these considerations, and by comparison with (4.18), the summation in equation (4.17) truncates at $n=2k-1$ when evaluating $c_k(z)$ for $k\ge 2$. Moreover, it turns out the last term in this truncated summation always vanishes due to the fact that it takes the form

$$-\frac{(2t)^{2k-1}}{(2k)!}(\mathcal{D}^{a_1}\mathcal{D}^{a_2}\ldots\mathcal{D}^{a_{2k-2}}G^{a_{2k-1}a_{2k}})K^{\alpha}{}_{\alpha a_1a_2\ldots a_{2k}}(z,t)\qquad k\ge 2 \tag{4.21}$$

and the moment is only ever required to leading order in its power series in t. To this order the moment is always totally symmetric in its spacetime indices, whereas G is antisymmetric. Consequently all such terms vanish[5], and when evaluating $c_k(z)$ the summation truncates at $n=2k-2$.

[4] By 'leading term' we mean the first (expected) non-trivial term, ie $K_{+}(z,t)$ has a leading term of order t^0.

[5] Alternatively, to this order the moment is proportional to the identity matrix in its group indices, and the coefficient therefore vanishes under integration by parts.

To compute $c_k(z)$ one must expand the set of moments which result from the summation in (4.17) to appropriate order in t. This is achieved through either direct expansion of the moment's exponential, or iteratively through the use of the identities

$$0 = \int \mathrm{d}\eta_+ \frac{\partial}{\partial k_b} \left(X_{A_1} \ldots X_{A_n} \mathrm{e}^{t\Delta} \right), \tag{4.22}$$

$$0 = \int \mathrm{d}\eta_+ \frac{\partial}{\partial \epsilon_\alpha} \left(X_{A_1} \ldots X_{A_n} \mathrm{e}^{t\Delta} \right) \tag{4.23}$$

and

$$\frac{d^m K_{A_1 \ldots A_n}(z,t)}{dt^m} = \int \mathrm{d}\eta_+ \, X_{A_1} \ldots X_{A_n} \Delta^m e^{t\Delta} . \tag{4.24}$$

These can be used to express the desired moment in terms of moments which are easier to compute. For greater detail we direct the reader appendix A which illustrates this procedure, or to the earlier work [4].

5. Results

5.1 Superfield form

The results produced directly by the calculational procedure employed here can always be vastly simplified since almost all possible combinations of covariant derivatives on field strengths are produced. Surprisingly, there are so many terms generated in c_4 that it is no longer practical to compute by hand. All results can be brought into their most compact form through integration by parts, the cyclic property of the trace, the equations of motion, the repetitive use of the commutation relations (2.6), and application of on-shell identities such as

$$\mathcal{D}_\alpha \mathcal{D}_\beta \bar{\mathcal{D}}_{\dot\alpha} \bar{W}_{\dot\beta} = 4\varepsilon_{\alpha\beta} \{ \bar{W}_{\dot\alpha}, \bar{W}_{\dot\beta} \} , \qquad \mathcal{D}_\alpha \bar{\mathcal{D}}_{\dot\alpha} \mathcal{D}_\beta W^\alpha = -4 \{ \bar{W}_{\dot\alpha}, W_\beta \} , \tag{5.1}$$

$$(\mathcal{D}^a \mathcal{D}_a W^\alpha) = [W^\beta, \mathcal{D}_\beta W^\alpha] , \tag{5.2}$$

the latter being established through equation (3.15).

Despite the eventual simplicity of the result, the computation of c_4 is very involved and requires a great deal of work. Unlike c_2 and c_3, c_4 is not manifestly real after simplification, but can be brought into such a form by using the identities (modulo integration by parts):

$$\mathrm{tr}_\mathcal{R} \Big((\mathcal{D}^a W^\alpha)(\mathcal{D}_a \bar{W}_{\dot\alpha}) \bar{W}^{\dot\alpha} W_\alpha \Big) = \mathrm{tr}_\mathcal{R} \Big((\mathcal{D}^a \bar{W}_{\dot\alpha})(\mathcal{D}_a W^\alpha) W_\alpha \bar{W}^{\dot\alpha} \Big) , \tag{5.3}$$

$$\begin{aligned} &\mathrm{tr}_\mathcal{R} \Big((\mathcal{D}^a W^\alpha)(\mathcal{D}_a W_\alpha) \bar{W}^2 + \\ &\quad + 2(\mathcal{D}_\alpha W^\beta) W^\alpha W_\beta \bar{W}^2 \Big) = \mathrm{tr}_\mathcal{R} \Big((\mathcal{D}^a \bar{W}_{\dot\alpha})(\mathcal{D}_a \bar{W}^{\dot\alpha}) W^2 + 2(\bar{\mathcal{D}}_{\dot\alpha} \bar{W}^{\dot\beta}) \bar{W}^{\dot\alpha} \bar{W}_{\dot\beta} W^2 \Big) . \end{aligned} \tag{5.4}$$

Finally, to this order the one-loop effective action is computed to be:

$$\begin{aligned} \Gamma^{(1)}_{\chi,\mathcal{R}} = {}& \frac{1}{16\pi^2} \ln\left(\frac{\mu}{m}\right) \int \mathrm{d}^6 z \, \mathrm{tr}_\mathcal{R}(W^2) - \frac{1}{128\pi^2 m^2} \int \mathrm{d}^8 z \, \mathrm{tr}_\mathcal{R}(c_2) - \\ & - \frac{1}{256\pi^2 m^4} \int \mathrm{d}^8 z \, \mathrm{tr}_\mathcal{R}(c_3) - \frac{1}{192\pi^2 m^6} \int \mathrm{d}^8 z \, \mathrm{tr}_\mathcal{R}(c_4) \end{aligned} \tag{5.5}$$

where

$$\mathrm{tr}_{\mathcal{R}}(c_2) = \frac{1}{3}\mathrm{tr}_{\mathcal{R}}(G^{ab}G_{ba}) = -\frac{1}{6}\mathrm{tr}_{\mathcal{R}}\left((\bar{\mathcal{D}}_{\dot\alpha}\bar{W}_{\dot\beta})(\bar{\mathcal{D}}^{\dot\beta}\bar{W}^{\dot\alpha}) + (\mathcal{D}^{\alpha}W^{\beta})(\mathcal{D}_{\beta}W_{\alpha})\right) \quad (5.6)$$

$$\mathrm{tr}_{\mathcal{R}}(c_3) = \frac{2}{15}\mathrm{tr}_{\mathcal{R}}(W^{\alpha}\bar{W}_{\dot\alpha}W_{\alpha}\bar{W}^{\dot\alpha} - 4W^2\bar{W}^2) \quad (5.7)$$

$$\begin{aligned}\mathrm{tr}_{\mathcal{R}}(c_4) = \frac{1}{105}\mathrm{tr}_{\mathcal{R}}\Big(&2(\mathcal{D}^{a}W^{\alpha})(\mathcal{D}_{a}\bar{W}_{\dot\alpha})W_{\alpha}\bar{W}^{\dot\alpha} - 6(\mathcal{D}^{a}W^{\alpha})(\mathcal{D}_{a}W_{\alpha})\bar{W}^2 - \\ &- 3(\mathcal{D}^{a}W^{\alpha})(\mathcal{D}_{a}\bar{W}_{\dot\alpha})\bar{W}^{\dot\alpha}W_{\alpha} + 18(\mathcal{D}_{\alpha}W^{\beta})W^{\alpha}W_{\beta}\bar{W}^2 + \\ &+ \frac{5}{2}(\mathcal{D}_{\alpha}W^{\beta})W^{\alpha}\bar{W}_{\dot\alpha}W_{\beta}\bar{W}^{\dot\alpha}\Big) + \text{c.c.}\end{aligned} \quad (5.8)$$

and c.c. denotes the complex conjugate.

Note that c_2 actually vanishes under integration by parts since on-shell $(\mathcal{D}_{\alpha}\mathcal{D}_{\beta}W_{\gamma}) = 0$, and therefore provides no contribution to the effective action.

5.2 Component form

The component form of the above expressions can be extracted through the usual techniques (for example see [10]). The the bosonic component of $c_3(z)$ is computed to be:

$$\frac{1}{30}\mathrm{tr}_{\mathcal{R}}(2F^{ab}F_{ab}F^{cd}F_{cd} + 3F^{ab}F^{cd}F_{ab}F_{cd} - 4F^{ab}F_{bc}F^{cd}F_{da} - 16F^{ab}F_{bc}F_{ad}F^{dc}), \quad (5.9)$$

where

$$\begin{aligned}\nabla_a &= \partial_a - \mathrm{i}A_a\,, & [\nabla_a, \nabla_b] &= -\mathrm{i}F_{ab} \\ F_{ab} &= \partial_a A_b - \partial_b A_a - \mathrm{i}[A_a, A_b]\,, & \nabla_c(F_{ab}) &= \partial_c F_{ab} - \mathrm{i}[A_c, F_{ab}].\end{aligned} \quad (5.10)$$

As will be shown, this agrees exactly with the component results given in [30] and [31].

Extraction and comparison of the bosonic component of $c_4(z)$ is complicated by the fact that in the non-abelian case there are many possible tensor structures which are not all independent. More generally, in addition to some identities which are dependent on the spacetime dimension, structures with a given number of contracted F's and ∇F's may be related through: the Bianchi identity, the equations of motion, integration by parts, the cyclic property of the trace over the gauge indices, and the non-abelian identity

$$[F_{ab}, F_{cd}] = 2\mathrm{i}\nabla_{[a}\nabla_{b]}F_{cd}\,. \quad (5.11)$$

An independent set of such tensor structures forms a basis, and different bases are used throughout the literature, since different calculational procedures naturally select different bases. In the case presented here the basis is almost completely determined by the use of superspace. In particular, in superspace and hence at the component level, both covariant derivatives act on adjacent field strengths and are always contracted with one another. Complete details regarding the basis used here and the transformation from any basis into it are given in appendix B.

The bosonic component of $c_4(z)$ in this basis is:

$$-\frac{1}{210}\mathrm{tr}_{\mathcal{R}}\Big(19(\nabla^e F^{ab})(\nabla_e F_{ab})F^{cd}F_{cd} + 11(\nabla^e F^{ab})(\nabla_e F^{cd})F_{ab}F_{cd} +$$

$$+ 13(\nabla^e F^{ab})(\nabla_e F^{cd})F_{cd}F_{ab} + 32(\nabla^e F^{ab})(\nabla_e F_{bc})F^{cd}F_{da} -$$
$$- 60(\nabla^e F^{ab})(\nabla_e F_{ca})F_{bd}F^{dc} + \frac{261\mathrm{i}}{5}F^{ab}F_{bc}F^{cd}F_{de}F^e{}_a -$$
$$- 89\mathrm{i}F^{ab}F_{bc}F^{cd}F^e{}_aF_{de} - 41\mathrm{i}F^{ab}F^{cd}F_{bc}F^e{}_aF_{de} - \frac{7\mathrm{i}}{5}F^{ab}F^{cd}F^e{}_aF_{bc}F_{de}\Big) \,. \tag{5.12}$$

6. Comparison with literature

To date the bosonic DeWitt-Seeley coefficients associated with scalars, vectors or spinors in the presence of non-abelian background Yang-Mills fields in arbitrary spacetime dimension have been separately computed to low order [30, 31, 32]. Since at the component level the action (3.3) corresponds to supersymmetric matter (a set of massive scalars and their fermionic superpartners) coupled to a background non-abelian supersymmetric Yang-Mills field, a non-trivial check of the results derived here is available.

From the tables in [31] and [32], one can assemble the total bosonic component of the DeWitt-Seeley coefficients associated with a theory possessing N_1 vectors, N_0 scalars and $N_{1/2}$ spinors all in the adjoint representation, coupled to a Yang-Mills background by using[6]

$$a_n^{\mathrm{tot}} = N_1 a_n(\Delta_1) + (N_0 - 2N_1)a_n(\Delta_0) - \frac{N_{1/2}}{\gamma}a_n(\Delta_{1/2}) \tag{6.1}$$

where $\gamma = 1, 2, 4$ for Dirac, Majorana and Majorana-Weyl spinors respectively, and $a_n(\Delta_s)$ ($s = 0, 1, \frac{1}{2}$) denotes the contribution generated by the presence of second order (scalar, vector, spinor) operators in the original action.

At the component level the starting action (3.3) contained two scalars and two Majorana-Weyl spinors in $D = 4$, so from [31, 32] and equation (6.1), one generates the following on-shell bosonic components of the DeWitt-Seeley coefficients:

$$\mathrm{tr}_{\mathrm{Ad}}(a_3) = 0 \tag{6.2}$$

$$\mathrm{tr}_{\mathrm{Ad}}(a_4) = -\frac{1}{240}\mathrm{tr}_{\mathrm{Ad}}\Big(2F^{ab}F_{ab}F^{cd}F_{cd} + 3F^{ab}F^{cd}F_{ab}F_{cd} - \\ - 4F^{ab}F_{bc}F^{cd}F_{da} - 16F^{ab}F_{bc}F_{ad}F^{dc}\Big) \tag{6.3}$$

$$\mathrm{tr}_{\mathrm{Ad}}(a_5) = \frac{1}{21}\frac{1}{5!}\mathrm{tr}_{\mathrm{Ad}}\Big(- 10(\nabla^e F^{ab})(\nabla_e F_{bc})F^{cd}F_{da} - 32(\nabla^e F^{ab})(\nabla_e F_{ca})F_{bd}F^{dc} + \\ + 8(\nabla^a F^{ef})(\nabla_b F_{ef})F^{ac}F_{cb} + \frac{1}{2}(\nabla^e F^{ab})F_{ab}(\nabla_e F^{cd})F_{cd} - \\ - 42(\nabla^e F^{ab})F_{da}(\nabla_e F_{bc})F^{cd} + 6(\nabla_b F^{ef})F^{cb}(\nabla^a F_{ef})F_{ac} + \\ + 6(\nabla^e F^{ab})(\nabla_e F_{ab})F^{cd}F_{cd} - \frac{19}{2}(\nabla^e F^{ab})(\nabla_e F^{cd})F_{ab}F_{cd} - \\ - 28(\nabla^a F^{de})(\nabla^b F_{ec})F_{ab}F^c{}_d\Big) + (F^5 \text{ terms})\,. \tag{6.4}$$

The vanishing of a_3 is non-trivial, and as indicated only the derivative terms of a_5 have so far been computed at the component level [30, 31, 32]. Inspection reveals immediate

[6] In the notation used in [31] and [32], $a_n = b_{2n}$

agreement, up to an overall numerical multiplicative constant, between a_3, a_4 and the bosonic components of c_2 and c_3 respectively. Taking into account the relationship between the coefficients c_n and a_n in superspace, equation (3.25), and restricting $\mathcal{R}$ to be the adjoint representation, exact agreement is found.

Comparison between a_5 and c_4 is far less trivial since the results for a_5 in the literature are expressed in a different basis, which is non-minimal in $D = 4$. After expressing a_5 in our basis (see appendix B), we find:

$$\begin{aligned}
\mathrm{tr}_{\mathrm{Ad}}(a_5) \;=\; & \frac{1}{21}\frac{1}{5!}\mathrm{tr}_{\mathrm{Ad}}\Big(19(\nabla^e F^{ab})(\nabla_e F_{ab})F^{cd}F_{cd} + 11(\nabla^e F^{ab})(\nabla_e F^{cd})F_{ab}F_{cd} + \\
& + 13(\nabla^e F^{ab})(\nabla_e F^{cd})F_{cd}F_{ab} + 32(\nabla^e F^{ab})(\nabla_e F_{bc})F^{cd}F_{da} - \\
& - 60(\nabla^e F^{ab})(\nabla_e F_{ca})F_{bd}F^{dc}\Big) + (F^5\,\text{terms})\,. \qquad (6.5)
\end{aligned}$$

Comparing with the bosonic component of c_4 in equation (5.12), and again taking into account equation (3.25), exact agreement of the derivative terms is found.

7. Discussion

We can now give the ingredients for the calculation of the one-loop effective action for an arbitrary $\mathcal{N} = 2$ SYM theory to order F^5. As previously noted, the effective action will be given by an appropriate linear combination of: (i) $\Gamma^{(1)}_{\chi,R\oplus R_c}$ and $\Gamma^{(1)}_{\chi,\mathrm{Ad}}$ given by (5.5; (ii) the logarithm of the superdeterminant of the background covariant operator in the quadratic part of the action for the vector superfield in the adjoint representation. The latter,

$$\frac{\mathrm{i}}{2}\ln \mathrm{sDet}(\Box_{\mathrm{v}} - \mathrm{M}^2)\,, \qquad (7.1)$$

where M is an infrared regulator and the background vector d'Alembertian, $\Box_{\mathrm{v}}$, is

$$\Box_{\mathrm{v}} = \mathcal{D}^a\mathcal{D}_a - W^\alpha\mathcal{D}_\alpha + \bar{W}_{\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha}\,, \qquad (7.2)$$

was derived in [4], and to order F^5 is:

$$\frac{\mathrm{i}}{2}\ln \mathrm{sDet}(\Box_{\mathrm{v}} - M^2) = \frac{1}{32\pi^2 M^4}\int \mathrm{d}^8 z\, \mathrm{tr}_{\mathrm{Ad}}(a_4^{(\mathrm{v})}) + \frac{1}{16\pi^2 M^6}\int \mathrm{d}^8 z\, \mathrm{tr}_{\mathrm{Ad}}(a_5^{(\mathrm{v})}) \qquad (7.3)$$

where[7]

$$\begin{aligned}
\mathrm{tr}_{\mathrm{Ad}}(a_4^{(\mathrm{v})}) \;=\; & \frac{1}{3}\mathrm{tr}_{\mathrm{Ad}}(2\,W^2\bar{W}^2 - W^\alpha\bar{W}_{\dot\alpha}W_\alpha\bar{W}^{\dot\alpha}) \qquad (7.4)\\
\mathrm{tr}_{\mathrm{Ad}}(a_5^{(\mathrm{v})}) \;=\; & \frac{1}{30}\mathrm{tr}_{\mathrm{Ad}}\Big((\mathcal{D}^a W^\alpha)(\mathcal{D}_a W_\alpha)\bar{W}^2 + (\mathcal{D}^a W^\alpha)(\mathcal{D}_a \bar{W}_{\dot\alpha})\bar{W}^{\dot\alpha}W_\alpha - \\
& - (\mathcal{D}^a W^\alpha)(\mathcal{D}_a \bar{W}_{\dot\alpha})W_\alpha\bar{W}^{\dot\alpha} - 3(\mathcal{D}_\alpha W^\beta)W^\alpha W_\beta\bar{W}^2 - \\
& - (\mathcal{D}_\alpha W^\beta)W^\alpha\bar{W}_{\dot\alpha}W_\beta\bar{W}^{\dot\alpha}\Big) + \text{c.c.} \qquad (7.5)
\end{aligned}$$

[7]Here the expression for $\mathrm{tr}_{\mathrm{Ad}}(a_5^{(\mathrm{v})})$ from [4] has been simplified through use of equation (5.2).

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Finally, it is worth pointing out that in [4], when computing the one-loop effective action for $\mathcal{N} = 4$ SYM (which is just (7.3)), we where able to push this method for calculating heat kernel coefficients to order F^6, whereas in the chiral case presented here, only to order F^5 with comparable difficulty. This is primarily due to the fact that in the $\mathcal{N} = 4$ case the associated heat kernel has the power series behaviour,

$$K_{\rm v}(z,t) = \frac{t^2 a_4^{(\rm v)}}{16\pi^2} + \frac{t^3 a_5^{(\rm v)}}{16\pi^2} + \cdots\,, \tag{7.6}$$

whereas the power series of the chiral kernel was given by equation (3.17). Consequently computing F^5 terms in $\mathcal{N} = 4$ SYM (ie $a_5^{(\rm v)}$), merely involved the computation of the second non-trivial coefficient, whereas one must compute the forth non-trivial coefficient to acquire the F^5 contributions in the chiral case.

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A. The method of computation

Here we illustrate through example the method of computation of coincidence limits of moments of heat kernels employed in this paper.

The power series arguments presented in section 4 showed that in computing $c_2(z)$ the summation in equation (4.17) truncates at $n = 2$. Consider the $n = 1$ contribution:

$$-\frac{t}{2!}\int {\rm d}\eta_+\; X^\alpha X_\alpha X^a[\Delta, X_a]\,{\rm e}^{t\Delta}\,. \tag{A.1}$$

Using the commutation relations (4.9) and (4.19) this reduces to

$$-\frac{t}{2!}\Big(Q^{\alpha\beta a}K_{\alpha\beta a}(z,t) + R^{\alpha ab}K_{\alpha ab}(z,t) + S^{\alpha\beta}K_{\alpha\beta}(z,t)\Big)\,, \tag{A.2}$$

where only the moments which contribute to $c_2(z)$ have been retained[8] and on-shell

$$Q^{\alpha\beta a} = {\rm i}\varepsilon^{\beta\alpha}(\sigma^a)_{\gamma\dot\gamma}\bar W^{\dot\gamma}W^\gamma - 2(\mathcal{D}^\alpha\mathcal{D}^a W^\beta) + 2\varepsilon^{\beta\alpha}(\mathcal{D}^b G^a{}_b) \tag{A.3}$$
$$R^{\alpha ab} = 4(\mathcal{D}^\alpha G^{ba}) \tag{A.4}$$
$$S^{\alpha\beta} = -2(\mathcal{D}^\alpha\mathcal{D}^a\mathcal{D}_a W^\beta) + \varepsilon^{\beta\alpha}G^{ba}G_{ab} \tag{A.5}$$
$$G_{ab} = [X_a, X_b] = [\mathcal{D}_a, \mathcal{D}_b]\,. \tag{A.6}$$

The remaining moments in (A.2) must now be expanded in a power series to the required order in t. In the current example this does not pose any additional difficulties since all moments are required only to leading order (as in equation (4.20)), which can easily

[8] For example from (4.18) and (4.20), one observes that terms like $tK(z,t)$, $tK_\alpha(z,t)$ and $tK_{ab}(z,t)$ etc will provide no contribution to $c_2(z)$.

be obtained simply by expanding the exponential in the moment into commutators. For example:

$$K_{\alpha ab}(z,t)=\int \mathrm{d}\eta_+\, X_\alpha X_a X_b e^{t\Delta}=\frac{1}{16\pi^2}\frac{1}{t^2}\eta_{ab}W_\alpha+\mathcal{O}(t^{-1})\,, \tag{A.7}$$

where the k integral has been performed (after Wick rotation to a euclidean metric).

However, if we wish to compute $c_3(z)$, such a moment would have been required to subleading order. In such a case one does not simply expand the exponential into commutators, since this is a very cumbersome method to evaluate most moments to other than leading order. Rather, it can be expressed in terms of moments which are easier to compute, or need only be computed to first order. Here this can be achieved through the use of the identity (4.23), in the form:

$$0=\int \mathrm{d}\eta_+\,\frac{\partial}{\partial\epsilon_\gamma}\left(X_\alpha X_\beta X_a X_b \mathrm{e}^{t\Delta}\right). \tag{A.8}$$

After contraction of γ and β, and using equation (4.13), this leads to an expression for $K_{\alpha ab}(z,t)$,

$$K_{\alpha ab}(z,t)=t\int \mathrm{d}\eta_+\, X_\alpha X_\beta X_a X_b \sum_{n=0}^{\infty}\frac{t^n}{(n+1)!}ad_\Delta^n(W^\beta)\,\mathrm{e}^{t\Delta}\,. \tag{A.9}$$

Examining the power series behaviour it can be seen that to compute $K_{\alpha ab}(z,t)$ to subleading order, this summation will truncate at $n=2$, but in doing so generates another series of moments which also need to be computed, ie

$$K_{\alpha ab}(z,t)=tW^\beta K_{\alpha\beta ab}(z,t)+\cdots\,. \tag{A.10}$$

In turn these moments may be expressed in terms of others using identities like (4.22), (4.23) or (4.24).

Proceeding iteratively in this fashion, when computing $c_k(z)$ one eventually establishes a small group of moments which need to be computed to particular order. This group of moments naturally arrange themselves into a hierarchy, where computing any moment in the hierarchy depends on the computation of other moments higher up in the hierarchy (for example $K_{\alpha ab}(z,t)$ depends on $K_{\alpha\beta ab}(z,t)$ in (A.10)). One can then proceed to compute, in systematic fashion, all moments within the hierarchy by starting at the top.

B. Change of basis

In this appendix we briefly describe the basis used in this paper, and outline the general transformation from any other basis into it. In particular the expression for a_5 in equation (6.4), is transformed into (6.5).

For simplicity the following notation is introduced (where the trace is over the gauge indices in some representation $\mathcal{R}$ of the gauge group):

$$s_{0,0}=\mathrm{tr}_{\mathcal{R}}(F^{ab}F_{bc}F^{cd}F_{de}F^e{}_a) \qquad s_{0,1}=\mathrm{tr}_{\mathcal{R}}(F^{ab}F_{bc}F^{cd}F^e{}_aF_{de})$$
$$s_{0,2}=\mathrm{tr}_{\mathcal{R}}(F^{ab}F^{cd}F_{bc}F^e{}_aF_{de}) \qquad s_{0,3}=\mathrm{tr}_{\mathcal{R}}(F^{ab}F^{cd}F^e{}_aF_{bc}F_{de})$$

$$
\begin{aligned}
s_{0,4} &= \mathrm{tr}_{\mathcal{R}}(F^{ab}F_{bc}F^{c}{}_{a}F^{de}F_{de}) & s_{0,5} &= \mathrm{tr}_{\mathcal{R}}(F^{ab}F^{de}F_{bc}F^{c}{}_{a}F_{de})\\
s_{1,0} &= \mathrm{tr}_{\mathcal{R}}((\nabla^e F^{ab})(\nabla_e F_{ab})F^{cd}F_{cd}) & s_{1,1} &= \mathrm{tr}_{\mathcal{R}}((\nabla^e F^{ab})(\nabla_e F^{cd})F_{ab}F_{cd})\\
s_{1,2} &= \mathrm{tr}_{\mathcal{R}}((\nabla^e F^{ab})(\nabla_e F^{cd})F_{cd}F_{ab}) & s_{1,3} &= \mathrm{tr}_{\mathcal{R}}((\nabla^e F^{ab})(\nabla_e F_{bc})F^{cd}F_{da})\\
s_{1,4} &= \mathrm{tr}_{\mathcal{R}}((\nabla^e F^{ab})(\nabla_e F_{ca})F_{bd}F^{dc}) & s_{1,5} &= \mathrm{tr}_{\mathcal{R}}((\nabla^e F^{ab})(\nabla_e F^{cd})F_{bc}F_{da})\\
s_{1,6} &= \mathrm{tr}_{\mathcal{R}}((\nabla^e F^{ab})F_{da}(\nabla_e F_{bc})F^{cd}) & s_{1,7} &= \mathrm{tr}_{\mathcal{R}}((\nabla^e F^{ab})F^{cd}(\nabla_e F_{ab})F_{cd})\\
s_{1,8} &= \mathrm{tr}_{\mathcal{R}}((\nabla^e F^{ab})F_{bc}(\nabla_e F^{cd})F_{da}) & s_{1,9} &= \mathrm{tr}_{\mathcal{R}}((\nabla^e F^{ab})F_{ab}(\nabla_e F^{cd})F_{cd})\\
s_{1,10} &= \mathrm{tr}_{\mathcal{R}}((\nabla^e F^{ab})F^{cd}(\nabla_e F_{cd})F_{ab}) & s_{1,11} &= \mathrm{tr}_{\mathcal{R}}((\nabla^e F^{ab})F^{dc}(\nabla_e F_{cb})F_{da})\\
s_{2,1} &= \mathrm{tr}_{\mathcal{R}}(F^{ab}(\nabla_a F^{cd})(\nabla^e F_{bc})F_{de}) & s_{2,3} &= \mathrm{tr}_{\mathcal{R}}((\nabla^a F^{ef})(\nabla_b F_{ef})F^{ac}F_{cb})\\
s_{2,4} &= \mathrm{tr}_{\mathcal{R}}((\nabla^a F^{ef})(\nabla_b F_{ef})F^{bc}F_{ca}) & s_{2,5} &= \mathrm{tr}_{\mathcal{R}}((\nabla^a F^{ef})F^{cb}(\nabla_b F_{ef})F_{ac})\\
s_{2,6} &= \mathrm{tr}_{\mathcal{R}}((\nabla_b F^{ef})F^{cb}(\nabla^a F_{ef})F_{ac}) & s_{2,7} &= \mathrm{tr}_{\mathcal{R}}((\nabla^b F^{cd})(\nabla_c F^{ea})F_{de}F_{ab})\\
s_{2,9} &= \mathrm{tr}_{\mathcal{R}}((\nabla_e F^{bc})(\nabla^a F^{de})F_{ab}F_{cd}) & s_{2,10} &= \mathrm{tr}_{\mathcal{R}}((\nabla^a F^{de})(\nabla^b F_{ec})F_{ab}F^{c}{}_{d})\\
s_{2,11} &= \mathrm{tr}_{\mathcal{R}}((\nabla_a F^{de})(\nabla^c F_{be})F^{ab}F_{cd}) & s_{2,12} &= \mathrm{tr}_{\mathcal{R}}((\nabla^a F^{de})(\nabla_e F^{cb})F_{ab}F_{cd})\\
s_{2,13} &= \mathrm{tr}_{\mathcal{R}}((\nabla_e F^{bc})F_{cd}(\nabla^d F^{ea})F_{ab}) & s_{2,14} &= \mathrm{tr}_{\mathcal{R}}((\nabla^a F^{cd})(\nabla^b F_{de})F^{e}{}_{c}F_{ab})\\
s_{2,15} &= \mathrm{tr}_{\mathcal{R}}((\nabla_a F^{cd})(\nabla^e F^{ab})F_{cb}F_{ed}) & s_{2,16} &= \mathrm{tr}_{\mathcal{R}}(F^{de}(\nabla_e F^{cb})F_{ab}(\nabla^a F_{cd}))\\
s_{2,17} &= \mathrm{tr}_{\mathcal{R}}(F_{ab}(\nabla_e F^{bc})F^{ed}(\nabla^a F_{dc})) & s_{2,18} &= \mathrm{tr}_{\mathcal{R}}(F_{ab}(\nabla_e F^{bc})F_{cd}(\nabla^a F^{de}))\,.
\end{aligned}
$$

This is not a complete list, and excludes terms which can obviously be reduced to a linear combination of the above (modulo integration by parts and the equations of motion). In $D = 4$ the following set provides a basis for all such possible tensor structures:

$$\{s_{0,0}, s_{0,1}, s_{0,2}, s_{0,3}, s_{1,0}, s_{1,1}, s_{1,2}, s_{1,3}, s_{1,4}, s_{2,3}\}\,, \tag{B.1}$$

which consists of four F^5 structures, five structures with contracted covariant derivatives acting on adjacent field strengths, and a single structure with two covariant derivatives which are not contracted. We demonstrate this below.

B.1 F^5 terms

One can readily establish that the six F^5 terms: $s_{0,0}$, $s_{0,1}$, $s_{0,2}$, $s_{0,3}$, $s_{0,4}$ and $s_{0,5}$, are not linearly independent in four dimensions by using the following σ matrix identity:

$$\mathrm{tr}(\sigma^a\tilde{\sigma}^b\sigma^c\tilde{\sigma}^d\sigma^e\tilde{\sigma}^f) = \mathrm{tr}(\sigma^b\tilde{\sigma}^a\sigma^f\tilde{\sigma}^e\sigma^d\tilde{\sigma}^c)\,. \tag{B.2}$$

This implies

$$\Big(\mathrm{tr}(\sigma^a\tilde{\sigma}^b\sigma^c\tilde{\sigma}^d\sigma^e\tilde{\sigma}^f)\mathrm{tr}(\tilde{\sigma}^g\sigma^h\tilde{\sigma}^i\sigma^j) + c.c\Big)\mathrm{tr}_{\mathcal{R}}(F_{ab}F_{cd}F_{ef}F_{gh}F_{ij} + F_{ab}F_{ef}F_{cd}F_{gh}F_{ij}) = 0\,, \tag{B.3}$$

which reduces to

$$s_{0,1} - 2s_{0,2} + s_{0,3} + \frac{1}{2}s_{0,4} + \frac{3}{2}s_{0,5} = 0\,. \tag{B.4}$$

Similarly one can use (B.2) to establish

$$\Big(\mathrm{tr}(\sigma^a\tilde{\sigma}^b\sigma^c\tilde{\sigma}^d\sigma^e\tilde{\sigma}^f)\mathrm{tr}(\tilde{\sigma}^g\sigma^h\tilde{\sigma}^i\sigma^j) + c.c\Big)\mathrm{tr}_{\mathcal{R}}(F_{ab}F_{cd}F_{gh}F_{ef}F_{ij} + F_{ab}F_{ef}F_{gh}F_{cd}F_{ij}) = 0\,, \tag{B.5}$$

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which becomes

$$s_{0,0} - 2s_{0,1} - s_{0,2} + \frac{3}{2}s_{0,4} - \frac{1}{2}s_{0,5} = 0\,. \qquad \text{(B.6)}$$

Equations (B.4) and (B.6) then allow two of the six F^5 structures to be expressed in terms of the other four. For example we choose to treat $s_{0,4}$ and $s_{0,5}$ as dependent:

$$s_{0,4} = -\frac{3}{5}s_{0,0} + s_{0,1} + s_{0,2} + \frac{1}{5}s_{0,3} \qquad \text{(B.7)}$$

$$s_{0,5} = \frac{1}{5}s_{0,0} - s_{0,1} + s_{0,2} + \frac{3}{5}s_{0,3}\,. \qquad \text{(B.8)}$$

B.2 D^2F^4 terms

Using the equations of motion, the Bianchi identity, integration by parts, the cyclic property of the trace and the identity

$$[F_{ab}, F_{cd}] = 2\mathrm{i}\nabla_{[a}\nabla_{b]}F_{cd}\,, \qquad \text{(B.9)}$$

each of the terms $s_{1,i}$ with $6 \leq i \leq 11$ can be expressed in the proposed basis:

$$s_{1,6} = -\,2\mathrm{i}s_{0,1} + 2\mathrm{i}s_{0,2} - s_{1,4} - s_{1,5} \qquad \text{(B.10)}$$

$$s_{1,7} = 4\mathrm{i}s_{0,5} - 2s_{1,1} = \frac{4\mathrm{i}}{5}s_{0,0} - 4\mathrm{i}s_{0,1} + 4\mathrm{i}s_{0,2} + \frac{12\mathrm{i}}{5}s_{0,3} - 2s_{1,1} \qquad \text{(B.11)}$$

$$s_{1,8} = -\,2\mathrm{i}s_{0,0} + 2\mathrm{i}s_{0,1} - 2s_{1,3} \qquad \text{(B.12)}$$

$$s_{1,9} = 4\mathrm{i}s_{0,4} - s_{1,0} - s_{1,2} = -\frac{12\mathrm{i}}{5}s_{0,0} + 4\mathrm{i}s_{0,1} + 4\mathrm{i}s_{0,2} + \frac{4\mathrm{i}}{5}s_{0,3} - s_{1,0} - s_{1,2} \qquad \text{(B.13)}$$

$$s_{1,10} = 4\mathrm{i}s_{0,4} - s_{1,0} - s_{1,2} = -\frac{12\mathrm{i}}{5}s_{0,0} + 4\mathrm{i}s_{0,1} + 4\mathrm{i}s_{0,2} + \frac{4\mathrm{i}}{5}s_{0,3} - s_{1,0} - s_{1,2} \qquad \text{(B.14)}$$

$$s_{1,11} = -\,2\mathrm{i}s_{0,1} + 2\mathrm{i}s_{0,2} - s_{1,4} - s_{1,5}\,. \qquad \text{(B.15)}$$

B.3 $D_aD_bF^4$ terms

Again using these properties one can generate the following independent equations:

$$s_{2,1} = \mathrm{i}s_{0,2} - \mathrm{i}s_{0,3} + \frac{1}{4}s_{1,1} - \frac{1}{2}s_{2,5} - s_{2,9} \qquad \text{(B.16)}$$

$$s_{2,9} = -\,\mathrm{i}s_{0,1} + \mathrm{i}s_{0,2} - s_{1,4} - \frac{1}{2}s_{2,3} + \frac{1}{2}s_{2,6} \qquad \text{(B.17)}$$

$$s_{2,4} = -\,\mathrm{i}s_{0,4} + \mathrm{i}s_{0,5} + \frac{1}{2}s_{1,2} - \frac{1}{2}s_{1,10} + s_{2,3} \qquad \text{(B.18)}$$

$$s_{2,5} = s_{2,6} \qquad \text{(B.19)}$$

$$s_{2,7} = s_{2,9} \qquad \text{(B.20)}$$

$$s_{2,13} = s_{2,18} = -\mathrm{i}s_{0,0} + \mathrm{i}s_{0,1} - s_{1,3} \qquad \text{(B.21)}$$

$$s_{2,12} = -\,s_{2,15} \qquad \text{(B.22)}$$

$$s_{2,12} = -\,s_{1,5} + s_{2,7} \qquad \text{(B.23)}$$

$$s_{2,10} = s_{2,14} = -s_{2,11} - s_{2,12} \qquad \text{(B.24)}$$

$$s_{2,10} = -\,s_{2,1} + s_{2,9} \qquad \text{(B.25)}$$

$$s_{2,16} = -\, s_{2,17} = \frac{\mathrm{i}}{2} s_{0,4} - \frac{\mathrm{i}}{2} s_{0,5} - s_{2,13} + s_{2,14}\,. \tag{B.26}$$

Furthermore, two more independent equations can be produced by using identity (B.2), for example:

$$\begin{aligned} &\Big(\mathrm{tr}(\sigma^a\tilde{\sigma}^b\sigma^c\tilde{\sigma}^d\sigma^e\tilde{\sigma}^f)\mathrm{tr}(\tilde{\sigma}^g\sigma^h\tilde{\sigma}^i\sigma^j) + \mathrm{c.c}\Big)\times \\ &\times\Big(\mathrm{tr}_{\mathcal{R}}((\nabla_b F_{cd})\nabla_a(F_{gh})F_{ij}F_{ef} - (\nabla_a F_{fe})\nabla_b(F_{gh})F_{ij}F_{dc})\Big) = 0\,, \end{aligned} \tag{B.27}$$

and

$$\begin{aligned} &\Big(\mathrm{tr}(\sigma^a\tilde{\sigma}^b\sigma^c\tilde{\sigma}^d\sigma^e\tilde{\sigma}^f)\mathrm{tr}(\tilde{\sigma}^g\sigma^h\tilde{\sigma}^i\sigma^j) + \mathrm{c.c}\Big)\times \\ &\times\Big(\mathrm{tr}_{\mathcal{R}}((\nabla_a F_{gj})\nabla_f(F_{hi})F_{bc}F_{ed} - (\nabla_b F_{gj})\nabla_c(F_{hi})F_{af}F_{de})\Big) = 0\,, \end{aligned} \tag{B.28}$$

which reduce to

$$s_{1,0} + s_{1,1} - 4s_{1,4} + 4s_{2,1} + 2s_{2,4} = 0 \tag{B.29}$$

and

$$s_{1,1} - s_{1,2} + 4s_{1,3} - 4s_{1,4} + 8s_{1,5} - 2s_{2,3} + 2s_{2,4} - 4s_{2,9} + 4s_{2,12} - 8s_{2,15} = 0 \tag{B.30}$$

respectively.

B.4 The basis

All tensor structures can now be expressed in the basis (B.1). Introducing the condensed notation,

$$\begin{aligned} \{a,b,c,d,e,f,g,h,i,j\} \equiv\ & a s_{0,0} + b s_{0,1} + c s_{0,2} + d s_{0,3} + e s_{1,0} + f s_{1,1} + \\ & + g s_{1,2} + h s_{1,3} + i s_{1,4} + j s_{2,3} \end{aligned} \tag{B.31}$$

we list below all terms expressed in this basis:

$$\begin{aligned} s_{0,4} &= \Big\{-\frac{3}{5}, 1, 1, \frac{1}{5}, 0, 0, 0, 0, 0, 0\Big\} \\ s_{0,5} &= \Big\{\frac{1}{5}, -1, 1, \frac{3}{5}, 0, 0, 0, 0, 0, 0\Big\} \\ s_{1,5} &= \Big\{2\mathrm{i}, -5\mathrm{i}, 0, -\mathrm{i}, \frac{3}{4}, \frac{3}{4}, \frac{3}{4}, 1, -3, 0\Big\} \\ s_{1,6} = s_{1,11} &= \Big\{-2\mathrm{i}, 3\mathrm{i}, 2\mathrm{i}, \mathrm{i}, -\frac{3}{4}, -\frac{3}{4}, -\frac{3}{4}, -1, 2, 0\Big\} \\ s_{1,7} &= \Big\{\frac{4\mathrm{i}}{5}, -4\mathrm{i}, 4\mathrm{i}, \frac{12\mathrm{i}}{5}, 0, -2, 0, 0, 0, 0\Big\} \\ s_{1,8} &= \Big\{-2\mathrm{i}, 2\mathrm{i}, 0, 0, 0, 0, 0, -2, 0, 0\Big\} \\ s_{1,9} = s_{1,10} &= \Big\{-\frac{12\mathrm{i}}{5}, 4\mathrm{i}, 4\mathrm{i}, \frac{4\mathrm{i}}{5}, -1, 0, -1, 0, 0, 0\Big\} \end{aligned}$$

$$
\begin{aligned}
s_{2,1} &= \left\{ -\mathrm{i}, 2\mathrm{i}, \mathrm{i}, 0, -\frac{1}{2}, -\frac{1}{4}, -\frac{1}{2}, 0, 1, -\frac{1}{2} \right\} \\
s_{2,4} &= \left\{ 2\mathrm{i}, -4\mathrm{i}, -2\mathrm{i}, 0, \frac{1}{2}, 0, 1, 0, 0, 1 \right\} \\
s_{2,5} = s_{2,6} &= \left\{ \mathrm{i}, -\mathrm{i}, -\mathrm{i}, -\mathrm{i}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 0, 0, 1 \right\} \\
s_{2,7} = s_{2,9} &= \left\{ \frac{\mathrm{i}}{2}, -\frac{3\mathrm{i}}{2}, \frac{\mathrm{i}}{2}, -\frac{\mathrm{i}}{2}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, 0, -1, 0 \right\} \\
s_{2,10} = s_{2,14} &= \left\{ \frac{3\mathrm{i}}{2}, -\frac{7\mathrm{i}}{2}, -\frac{\mathrm{i}}{2}, -\frac{\mathrm{i}}{2}, \frac{3}{4}, \frac{1}{2}, \frac{3}{4}, 0, -2, \frac{1}{2} \right\} \\
s_{2,11} &= \left\{ 0, 0, 0, 0, -\frac{1}{4}, 0, -\frac{1}{4}, 1, 0, -\frac{1}{2} \right\} \\
s_{2,12} = -s_{2,15} &= \left\{ -\frac{3\mathrm{i}}{2}, \frac{7\mathrm{i}}{2}, \frac{\mathrm{i}}{2}, \frac{\mathrm{i}}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -1, 2, 0 \right\} \\
s_{2,13} = s_{2,18} &= \left\{ -\mathrm{i}, \mathrm{i}, 0, 0, 0, 0, 0, -1, 0, 0 \right\} \\
s_{2,16} = -s_{2,17} &= \left\{ \frac{21\mathrm{i}}{10}, -\frac{7\mathrm{i}}{2}, -\frac{\mathrm{i}}{2}, -\frac{7\mathrm{i}}{10}, \frac{3}{4}, \frac{1}{2}, \frac{3}{4}, 1, -2, \frac{1}{2} \right\}
\end{aligned}
\tag{B.32}
$$

Finally, in this basis we find that the derivative terms of a_5, given in equation (6.4), become

$$
a_5 = \frac{1}{21}\frac{1}{5!}\left\{ \frac{234\mathrm{i}}{5}, -32\mathrm{i}, -74\mathrm{i}, -\frac{168\mathrm{i}}{5}, 19, 11, 13, 32, -60, 0 \right\}, \tag{B.33}
$$

which, ignoring F^5 terms, is equation (6.5).

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